Characteristic and necessary minutiae in fingerprints

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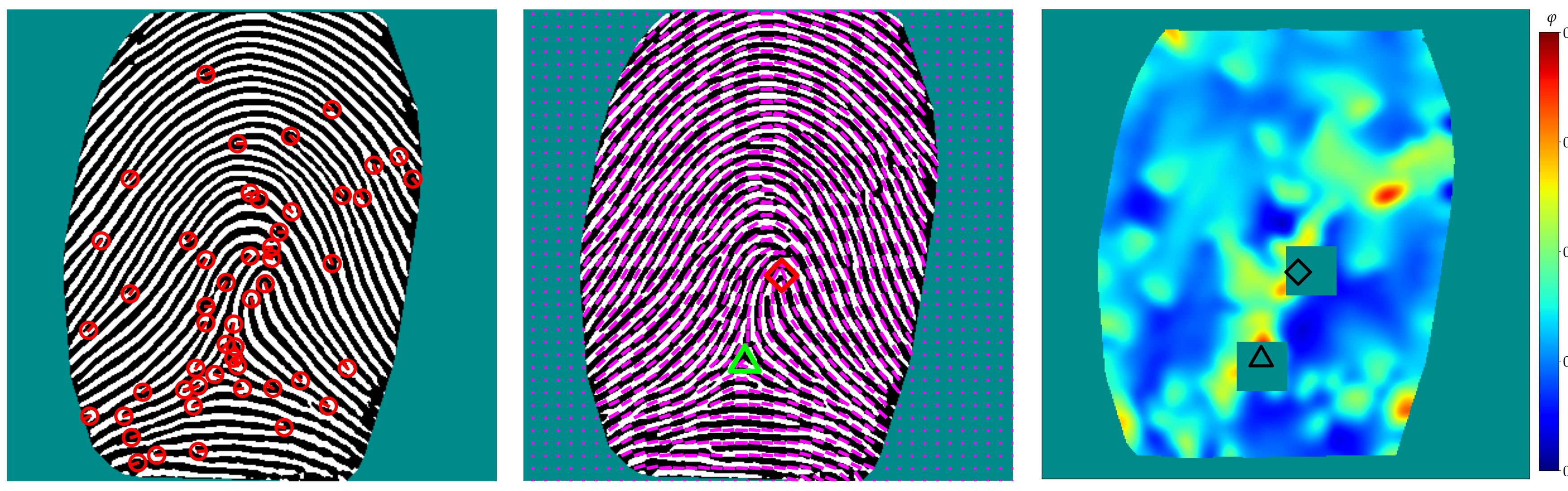
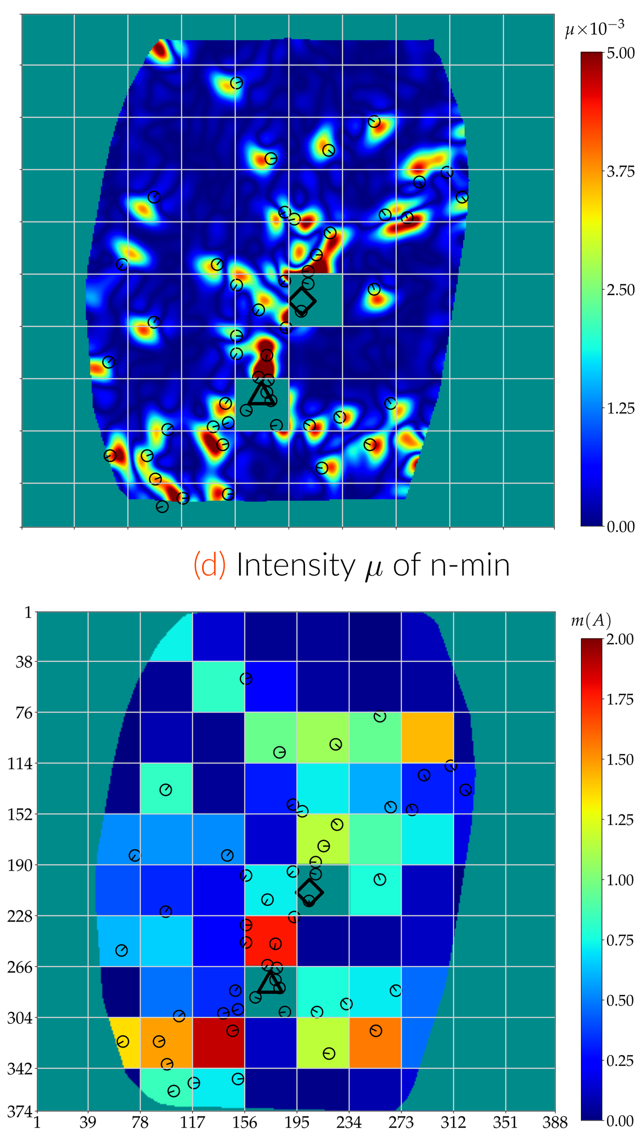
Features in fingerprints

- Level 1 (global): region of interest $\mathfrak{X} \subseteq \mathbb{R}^2$, orientation field (OF) $\vec{F} : \mathfrak{X} \to \mathbb{S}^1$, ridge frequency (RF) $\varphi : \mathfrak{X} \to \mathbb{R}_+$,
- Level 2 (local): minutiae $\zeta = \{z_1, \dots, z_n\} \subseteq \mathfrak{X}$ (ridge endings/ bifurcations)



(a) Minutiae pattern (b) Orientation field (c) Ridge frequency



(d) Intensity μ of n-min

(e) $m(A)$: number of n-min in patches A

Figure 1. Fingerprint features

Definition (Necessary minutiae number)

For simply connected, compact $A \subseteq \mathfrak{X}$ with piecewise smooth boundary ∂A with outer unit normal $\vec{n} : \partial A \to \mathbb{S}^1$, not containing singularities of $\vec{F} \in \mathcal{C}^2(A \to \mathbb{S}^1)$ and $\varphi \in \mathcal{C}^2(A \to \mathbb{R}_+)$ we call

$$m(A) := \left| \int_{\partial A} \varphi(z) \left\langle \vec{F}(z), \vec{n}(z) \right\rangle \, \mathrm{d}z \right|$$

the *number of geometrically necessary minutiae* (n-min) in A.

Theorem (Minutiae divergence formula)

Let $A \subseteq \mathfrak{X}$ be as above and, moreover, star-shaped w.r.t. $z_0 \in \mathfrak{X}$. Then

$$m(A) = \left| \underbrace{\varphi(z_0) \operatorname{div} \vec{F}(z_0)}_{\text{OF divergence}} + \underbrace{\left\langle \nabla\varphi(z_0), \vec{F}(z_0) \right\rangle}_{\text{RF divergence}} \right| \cdot |A| + o(|A|)$$

as $\sup_{z \in A} \|z - z_0\| \to 0.$

We call

$$\mu(z_0) := \left| \varphi(z_0) \operatorname{div} \vec{F}(z_0) + \left\langle \nabla\varphi(z_0), \vec{F}(z_0) \right\rangle \right|$$

the *intensity of necessary minutiae* at $z_0 \in \mathfrak{X}$.

Existence of random minutiae

Analysis of 20 high quality fingerprints from [2] suggests existence of additional random minutiae, $\approx 10^{-4}$ per pixel at a resolution of 500dpi.

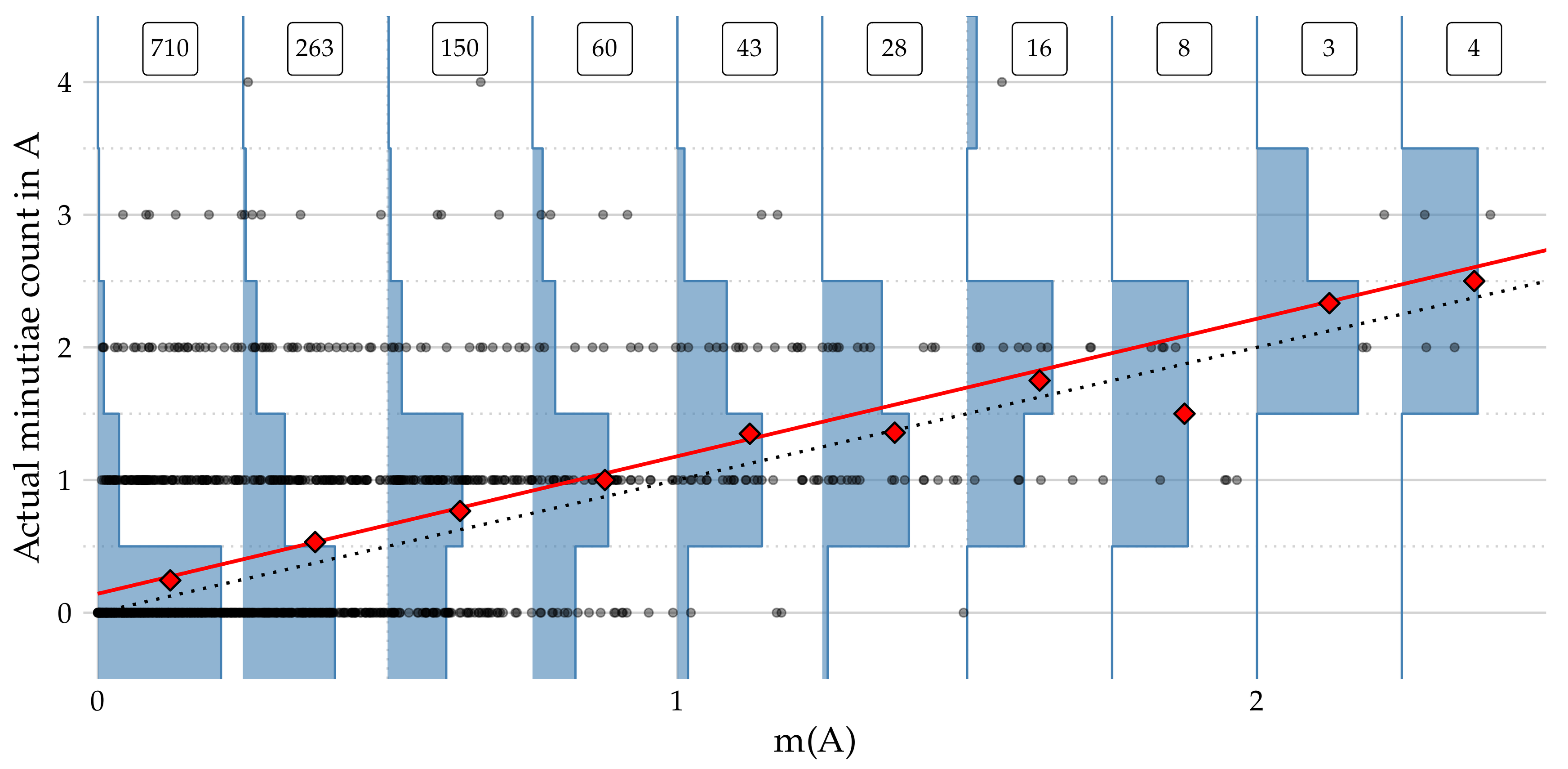


Figure 2. Poisson regression actual min-count vs. $m(A)$ for the patches from the depicted grid partition.

Superposition model

Necessary minutiae H

Assume $\mathrm{H} \sim \mathrm{StraussHard}(\beta\mu, \gamma, r, R)$ with $\beta > 0$, $\gamma \in (0,1)$ and known interaction distances $0 < r < R$:

$$f_{\beta,\gamma}(\eta) = c(\beta,\gamma) \left(\prod_{z_i \in \eta} \beta\mu(z_i) \right) \gamma^{s_R(\eta)} \mathbf{1}\left(d_{\min}(\eta) > r\right), \quad s_R(\eta) := \sum_{z_i \neq z_j \in \eta} \mathbf{1}\left(\|z_i - z_j\| \leq R\right),$$

where $\mu : \mathfrak{X} \to \mathbb{R}_+$ is the intensity of necessary minutiae.

Random minutiae Ξ

Assume $\Xi \sim \mathrm{Poisson}(\lambda)$ with $\lambda > 0$:

$$g_\lambda(\xi) = \mathrm{e}^{(1-\lambda)|\mathfrak{X}|} \left(\prod_{z_i \in \xi} \lambda \right) = \mathrm{e}^{(1-\lambda)|\mathfrak{X}|} \lambda^{|\xi|}.$$

Parameter and sample space

- data $\zeta = \eta \dot{\cup} \xi \sim \mathrm{Z} := \mathrm{H} \dot{\cup} \Xi$
- parameters $\theta = (\beta, \gamma, \lambda) \in \Theta := \mathbb{R}_{++} \times (0,1) \times \mathbb{R}_{++}$, latent vector $W \in \{0,1\}^n$ with $W_i = \mathbf{1}(z_i \in \eta)$

Literature overview

- [5] – separation of homogeneous Strauss and Poisson processes using MCMC (poor mixing)
- [4] – setting as in [5], does not account for dependencies in W

Model analysis

- Poisson process: independent distribution of points, high variance of point numbers in regions of high intensity $\rightsquigarrow$ large clusters possible, cf. Figure 3
- Strauss-hard core: total repulsion at inter-ridge distance and inhibition also at larger scales (regularisation of variance of point numbers in regions of large intensity); too few points at distance 25-35 pixels, cf. Figure 4 $\rightsquigarrow$ additional Poisson points

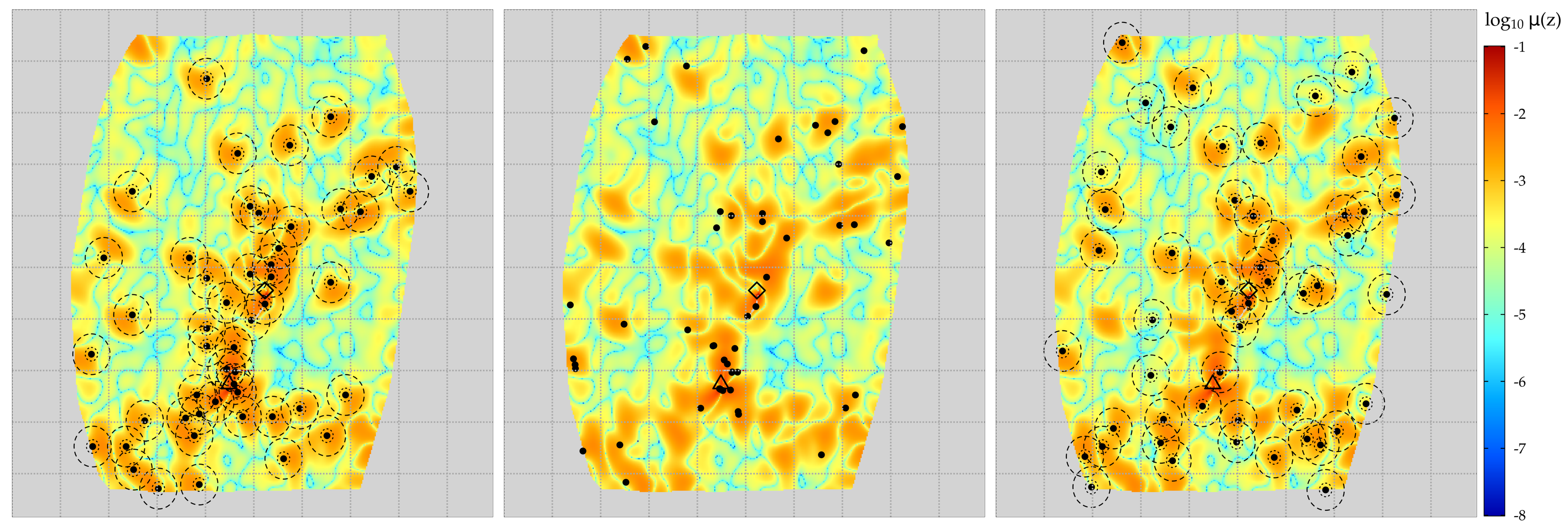


Figure 3. Real (left) and simulated minutiae patterns from Poisson (middle) and Strauss-hard core (right) processes. The intensity field μ as heat map in the background has been scaled s.t. all point patterns have the same number of points.

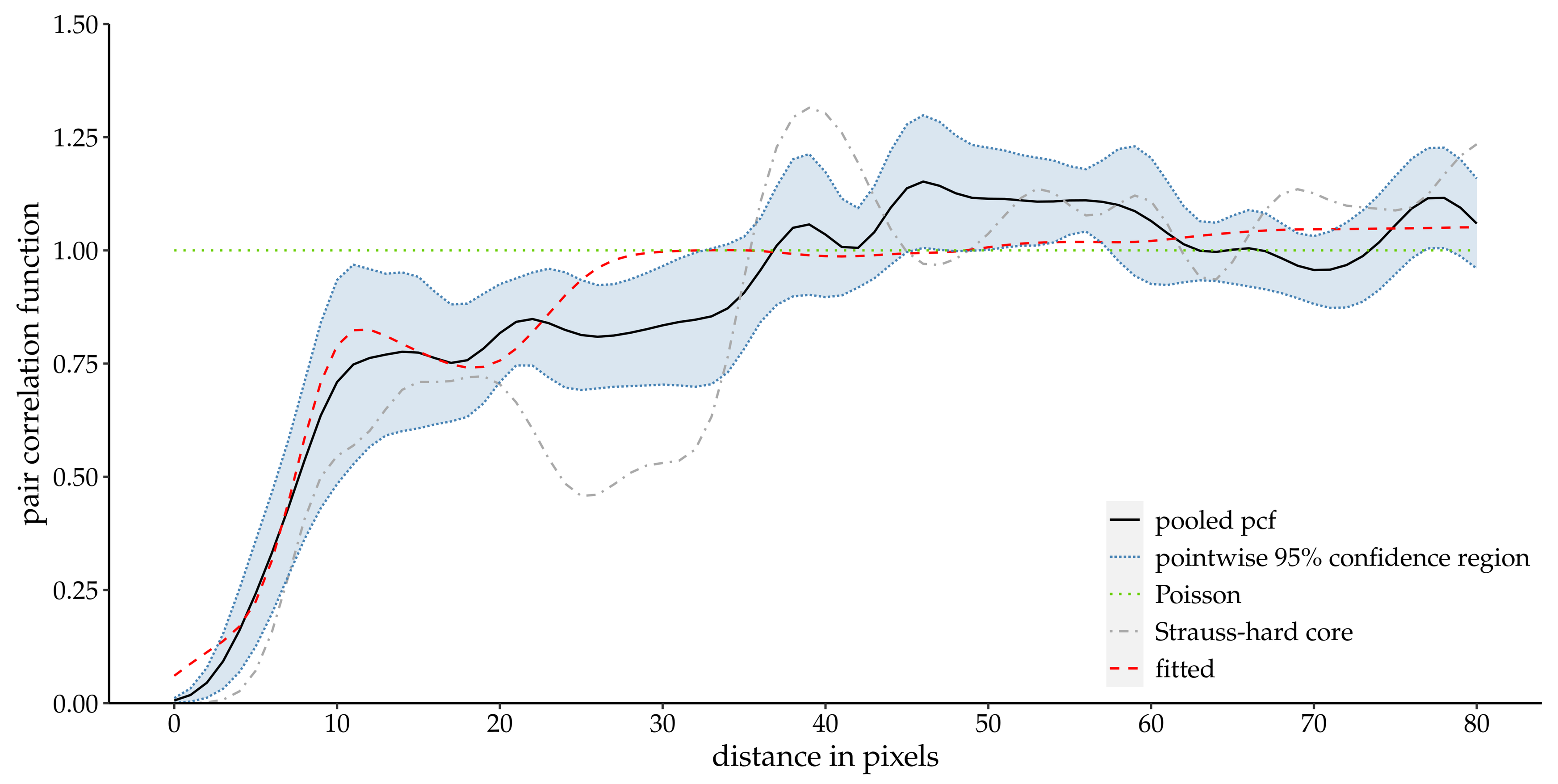


Figure 4. Pooled pair correlation function for 20 high quality fingerprints from Figure 2, [2].

The Minutiae Separating algorithm (MiSeal) [6]

- frequentist parameter estimation computationally very expensive (intractable normalising constants); no inference about W possible $\rightsquigarrow$ Bayesian approach
- MCMC for sampling from the posterior distribution $\pi((\theta, W) \mid \zeta)$
- alternating updates of θ and W (random scan Gibbs sampling)
- auxiliary variable method for (β, γ)-update using auxiliary Strauss point patterns (doubly MCMC)

Random minutiae are characteristic

Consider fingerprints from monozygotic twins exhibiting similar OF and RF. For the minutiae patterns $\zeta^{(i)}$, $i = 1, 2$, apply

1. Draw sample $W^{(i)} \sim \pi_i$ from the posterior $\pi_i = \pi(W \mid \zeta^{(i)})$ of W; let $r^{(i)}$ be the number of random minutiae in $\zeta^{(i)}$.
2. Delete from $\zeta^{(i)}$ the minutiae with $W_j^{(i)} = 0$; obtain new template $\zeta^{(i,n)}$ containing only necessary minutiae.
3. Draw uniformly $r^{(i)}$ minutiae from $\zeta^{(i)}$, delete them from $\zeta^{(i)}$ to obtain $\zeta^{(i,r)}$ (same number of minutiae as $\zeta^{(i,n)}$).
4. Compute matching scores $S^{(n)} := S(\zeta^{(1,n)}, \zeta^{(2,n)})$, $S^{(r)} := S(\zeta^{(1,r)}, \zeta^{(2,r)})$ using minutiae cylinder code [1].

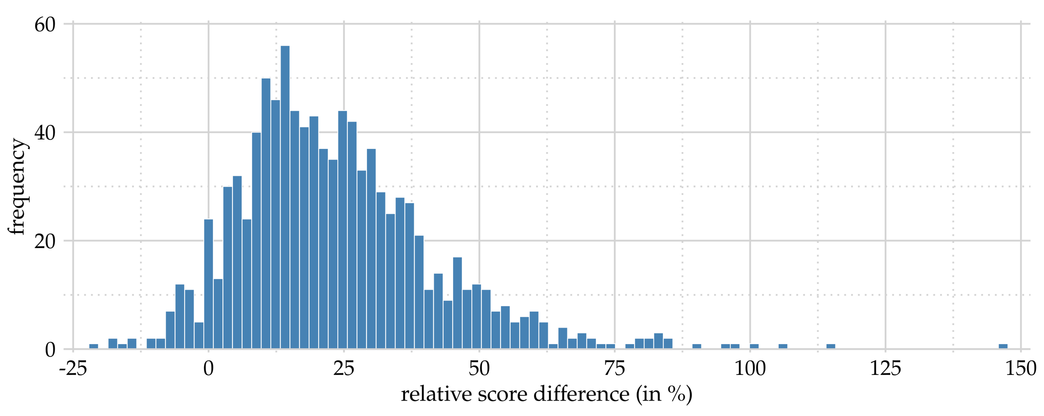


Figure 5. Histogram of relative score differences $\left(S^{(n)} - S^{(r)}\right)/S^{(r)}$ (in %). Overall 93.6% of these distances are positive and the average difference is 23.7%.

Figure 6. Fingerprints from a pair of monozygotic twins [3] with almost identical OF and RF.

Conclusion: Fingerprints are (usually) getting more similar by deleting only the random (*characteristic*) minutiae compared to random deletion of the same number of minutiae.

References

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